The Response of Hours to a Technology Shock: a Two–Step Structural VAR Approach

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Keywords: SVARs, long–run restriction, technology shocks, hours worked

JEL Class.: C32, E32

January, 2009

*Address: GREMAQ–Université de Toulouse I, manufacture des Tabacs, bât. F, 21 allée de Brienne, 31000 Toulouse. We would like to thank an anonymous referee and the editor P. Evans, as well as J. Campbell, F. Collard, M. Dupaigne, M. Eichenbaum, J. Galí, L. Gambetti, S. Grégoir, A. Kurman, J. Matheron, F. Pelgrin, L. Phaneuf, F. Portier, H. Uhlig, R. Vigfusson and E. Wasmer for valuable remarks. This paper has benefited from helpful discussions during presentations at CIRANO Workshop on *Structural VARs*, UQAM seminar, *Macroeconomic Workshop* (Aix/Marseille), Laser Seminar (Montpellier), HEC-Lausanne seminar, HEC-Montréal seminar, Université Laval seminar and AMeN workshop (Barcelona). The traditional disclaimer applies. The views expressed herein are those of the authors and not necessary those of the Banque de France.

Abstract

The response of hours to a technology shock is a controversial issue in macroeconomics. Part of the difficulty lies in that the estimated response is sensitive to the specification of hours in SVARs. This paper uses a simple two-step approach to consistently estimate the response of hours. The first step considers a SVAR model with a relevant stationary variable, but excluding hours. Given a consistent estimate of technology shocks in the first step, the response of hours to this shock is estimated in a second step. Simulation experiments from an estimated DSGE show that this approach outperforms standard SVARs. When applied to US data, the two-step approach predicts a short-run decrease followed by a hump-shaped positive response. This result is robust to other specifications and data.

Introduction

The response of hours to a technology shock is the subject of many controversies in quantitative macroeconomics. The contributions of Galí (1999), Basu, Fernald and Kimball (2006) and Francis and Ramey (2005) show that the short–run response of hours worked to a technology shock is significantly negative in the US economy. Galí (1999) and Francis and Ramey (2005) obtain this result using a Structural Vector Autoregression (SVAR) of labor productivity growth and hours in first difference (DSVAR) with long–run restrictions (see Blanchard and Quah, 1989). Basu, Fernald and Kimball (2006) use a direct measure of aggregate technology change, controlling for imperfect competition, varying utilization of factors and aggregation effects, and find that hours fall significantly on impact after a technology improvement. Moreover, Galí (1999, 2004) shows that the level of hours significantly decreases in the short run in all G7 countries and the euro area as a whole, with the exception of Japan. These results are in contradiction with Christiano, Eichenbaum and Vigfusson (2004). Using a SVAR with a level specification of hours (LSVAR), they find a positive and hump–shaped response of hours after a technology shock. Moreover, they show that the LSVAR specification encompasses the DSVAR specification.

The specification of hours in level or in difference appears to be the core issue of the controversies. Galí (1999), and Galí and Rabanal (2004) and Christiano, Eichenbaum and Vigfusson (2004) perform various unit root tests, but it becomes hard to obtain clear–cut evidence in favor of level or difference specification. Furthermore, recent contributions proceeding with simulation experiments point out that the specification of hours in SVARs using long–run restrictions can alter significantly the estimated effect of a technology shock on hours. For example, Chari, Kehoe and McGrattan (2008) simulate a business cycle model estimated by Maximum Likelihood on US data with multiple shocks. They show that the DSVAR specification leads to a negative response of hours under a RBC model in which hours respond positively. As pointed out by Christiano, Eichenbaum and Vigfusson (2004), the DSVAR specification may induce strong distortions if hours worked are stationary in level.

In this paper, we use a simple method that allows us to consistently estimate technology shocks and thus the responses of hours to a technology improvement. In contrast to existing LSVAR and DSVAR specifications, we choose to exclude hours worked series from SVARs to identify technology shocks.[1] The proposed approach consists in the following two steps. In a first step, a SVAR model with long–run restriction that includes well-chosen covariance stationary variables allows to properly identify the technology shock series. Among these variables, the consumption to output ratio seems to be a promising candidate. Two reasons motivate this choice. First, as argued by Cochrane (1994), this ratio may help to better predict the permanent and transitory components of output. Indeed, using a simple permanent income argument, permanent (technology) shocks can be separated from other (non–permanent and non–technology) shocks because these latter do not modify the consumption. The joint observation of output growth and consumption to output ratio allows then the econometrician to properly identify permanent and transitory shocks. Second, both from the simulations of a DSGE model and the actual data, we obtain that the consumption to output ratio displays less persistence than hours. When a SVAR model with long–run restrictions includes variables characterized by a highly persistent process (typically hours with the level specification), the identification of the responses of hours to technology shocks can be seriously disturbed. Gospodinov (2008) using a near-unit root process for the hours and Christiano, Eichenbaum and Vigfusson (2004) using a unit root process have shown that the LSVAR specification for such highly persistent processes leads an inconsistent estimator of the technology shocks. With respect to this result, a less persistent variable such as consumption to output ratio should improve the identification of the technology shocks. Moreover, the specification of this ratio is not subject to controversies in quantitative macroeconomics and the cointegration between consumption and output is usually imposed in SVARs (see Cochrane, 1994, Christiano, Eichenbaum and Vigfusson, 2004, Francis and Ramey, 2005, King, Plosser, Stock and Watson, 1991, among others).

In the second step, the Impulse Response Functions (IRFs) of hours at different horizons are obtained by a simple regression of hours on the estimated technology shocks for different lags. In this latter step, according to the debate about the right specification of hours, we consider hours worked in level and in difference in this regression. We obtain that the specification of hours does not matter either in the identification step and the estimation step. Our method can be seen as a combination of a SVAR approach in the line of Blanchard and Quah (1989), Galí (1999) and Christiano, Eichenbaum and Vigfusson (2004) and the regression equation used by Basu, Fernald and Kimball (2006) in their growth accounting exercise. Our approach is also related to the paper of Francis and Ramey (2008) in which they construct a corrected measure of hours. When low frequency movements are removed from hours, they find that both level and first difference specifications in SVAR yield to a decline in hours. This is what we obtain with the two–step approach both from simulated and actual data. A key advantage of our approach is its simplicity because it is not necessary to compute neither a corrected measure of hours (as in Francis and Ramey, 2008) or a proper measure of Total Factor Productivity (TFP) (as in Basu, Fernald and Kimball, 2006). Moreover, our empirical strategy can be simply applied to a variety of data which display high degree of serial correlation.

We assess our proposed approach and compare it to SVARs (LSVAR and DSVAR models) using artificial data obtained from the simulation of a DSGE model. Using US quarterly data, we first estimate by maximum likelihood a DSGE model with real frictions, *i.e.* habits in consumption and investment adjustment costs. It should be noted that our estimation strongly rejects a frictionless version of the model. The estimated model leads to a decrease in hours worked in the short run, because of strong real frictions. We then simulate the model and compare the estimated dynamic responses of hours under the different approaches. Our results show that the two step approach outperforms LSVARs and DSVARs. Our findings suggest that the consumption to output ratio helps significantly to separate permanent from transitory components in labor productivity.

We then apply the two–step approach to US data. We obtain that hours worked decrease significantly in the short–run after a positive technology shock but display a positive hump–shaped response. Contrary to SVARs, the specification of hours in the second step does not matter a lot. Our results are in line with previous empirical findings which show that hours fall on impact (see Galí, 1999, Basu, Fernald and Kimball, 2006, Francis and Ramey, 2005, 2008) and display a positive hump pattern during the subsequent periods (see Christiano, Eichenbaum and Vigfusson, 2004 and Vigfusson, 2004). So, our approach allows to bridge the gap between the LSVAR and DSVAR specifications. These results appear robust to the sample period considered, measures of hours and output, bivariate VARs, relevant larger VARs and breaks in labor productivity. Interestingly, the results obtained in all cases are in accordance with the simulation experiments: the level and difference specifications of hours provide similar IRFs in all our estimations, the level specification of hours delivers uninformative IRFs characterized by wide confidence intervals, the dynamic responses when hours are taken in first difference in the second step are precisely estimated.

The paper is organized as follows. In a first section, we present our two-step approach. Section 2 presents simulation experiments from an estimated DSGE model of the US economy. Section 3 is devoted to the exposition of the empirical results. The last section concludes.

1 The Two-Step Approach

The goal of our approach is to accurately identify technology shocks in a first step using adequate covariance–stationary variables in the VAR model. A large part of the performance of the two-step approach depends on the time series properties of these variables, which can be interpreted as instruments allowing to estimate with more precision the true technology shocks.

The objective of the first step is then to include a set of variables in the SVAR model to properly identify the technology shocks series. Among these variables, a promising candidate is the log of consumption to output ratio. There is both structural and empirical evidence that supports the selection of this variable.

First, following Cochrane (1994), we argue that the consumption to output ratio contains useful econometric information to disentangle the permanent to the transitory component. Indeed, this ratio helps to identify transitory shocks as those that have no effect on consumption. The argument of Cochrane (1994) is based on a structural interpretation using a simple permanent income model. This model implies that consumption is a random walk and that consumption and total income are cointegrated. Consequently, it follows from the intertemporal decisions on consumption that any shock to aggregate output that leaves consumption constant is necessarily a transitory shock. The joint observation of output growth and the log of consumption to output ratio allows then the econometrician to decompose aggregate shocks into permanent and transitory shocks, as perceived by consumers.

Second, as shown from simulations experiments (see section 2) and actual data (see section 3), the unit root can be rejected for this ratio at a conventional level and the empirical autocorrelation function indicate a less persistent process than the one of hours. Gospodinov (2008) using a near-unit root process for the hours and Christiano, Eichenbaum and Vigfusson (2004) using an exact unit root process show that a SVAR model which includes such highly persistent processes leads to a weak instrument problem. This weak instrument problem implies that technology shocks and their impacts are inconsistently estimated. Consequently, the introduction of a less persistent variable in the VAR, as the consumption-output ratio, should improve the identification of the technology shocks by avoiding the weak instrument problem. The impact of these shocks on the variable of interest (hours worked) is evaluated in the second step. To do so, hours are projected in level and in difference on the identified technology shocks series. In the applications, we also consider in the first step larger SVARs that have been used in the relevant literature (see for example, Galí, 1999, Francis and Ramey, 2005a and Christiano, Eichenbaum and Vigfusson, 2004) to check the robustness of our two-step strategy. We now present in more details the two-step approach.

Step 1: *Identification of technology shocks*

Consider a VAR(p) model which includes productivity growth $\Delta(y_t - h_t)$ and consumption to output ratio $c_t - y_t$ (in logs).[2]:

$$X_t = \sum_{i=1}^{p} B_i X_{t-i} + \varepsilon_t \tag{1}$$

where $X_t = (\Delta(y_t - h_t), c_t - y_t)'$ and $\varepsilon_t = (\varepsilon_{1,t}, \varepsilon_{2,t})'$ with $\mathrm{E}(\varepsilon_t \varepsilon_t') = \Sigma$. Without loss of generality, we omit a constant term in (1). Under usual conditions, this VAR(p) model admits a VMA(∞) representation[3]

$$X_t = C(L)\varepsilon_t$$

where $C(L) = (I_2 - \sum_{i=1}^{p} B_i L^i)^{-1}$ and L is the backshift operator. The structural VMA(∞) representation is given by

$$X_t = A(L)\eta_t$$

where $\eta_t = (\eta_t^T, \eta_t^{NT})'$. η_t^T is period t technology shock, whereas η_t^{NT} is period t composite non–technology shock.[4] By normalization, these two orthogonal shocks have zero mean and unit variance. The identifying restriction implies that the composite non–technology shock has no long–run effect on labor productivity. This means that the upper triangular element of $A(L)$ in the long run must be zero, *i.e.* $A_{12}(1) = 0$. In order to uncover this restriction from the estimated VAR(p) model in equation (1), the matrix $A(1)$ is obtained by the Choleski decomposition of $C(1)\Sigma C(1)'$. The structural shocks are then directly deduced up to a sign restriction by

$$\begin{pmatrix} \eta_t^T \\ \eta_t^{NT} \end{pmatrix} = C(1)^{-1}A(1)\begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix}$$

Step 2: *Estimation of the response of hours to a technology shock*

The structural infinite moving average representation for hours worked as a function of the technology shock and the composite non–technology shock[5] is given by:

$$h_t = a_1(L)\eta_t^T + a_2(L)\eta_t^{NT}. \tag{2}$$

The coefficient $a_{1,k}$ ($k \geq 0$) measures the effect of the technology shock at lag k on hours worked, *i.e.* $a_{1,k} = \partial h_{t+k}/\partial \eta_t^T$.

The identifying restriction in Step 1 implies that non–technology shocks are orthogonal to technology shocks by construction, *i.e.* $\mathrm{E}(\eta_{t-i}^T, \eta_{t-j}^{NT}) = 0 \ \ \forall\, i,j$ and that the technology and non–technology shocks are serially uncorrelated which implies $\mathrm{E}(\eta_t^T, \eta_{t-i}^T) = 0$ and $\mathrm{E}(\eta_t^{NT}, \eta_{t-i}^{NT}) = 0 \ \ \forall i \neq 0$. These properties allow us to obtain consistent estimates of the dynamic responses.

According to the debate on the right specification of hours worked, we examine two specifications to measure the effect of a technology shock. In the first specification, hours are projected in level on the identified technology shocks while in the second specification, hours are projected in difference.

Let us now examine in more details both specifications. In the first one, the log of hours worked is regressed on the current and past values of the identified technology shocks $\widehat{\eta}_t^T$ in the first-step:

$$h_t = \sum_{i=0}^{q} \theta_i \widehat{\eta}_{t-i}^T + \nu_t \tag{3}$$

where $q < +\infty$ and $\hat{\eta}_t^T$ denotes the estimated technology shocks obtained from the SVAR model in the first step. ν_t is an error term that accounts for non–technology shocks and the remaining technology shocks. A standard OLS regression provides the estimates of the population responses of hours to the present and lagged values of the technology shocks, namely: $\widehat{a}_{1,k} = \widehat{\theta}_k$.

The log of hours worked is also regressed in first difference on the current and past values of the identified technology shocks. The response to a technology shock is now estimated from the regression:

$$\Delta h_t = \sum_{i=0}^{q} \tilde{\theta}_i \widehat{\eta}_{t-i}^T + \tilde{\nu}_t. \tag{4}$$

As hours are specified in first difference, the estimated response at horizon k is obtained from the cumulated OLS estimates: $\widehat{\tilde{a}}_{1,k} = \sum_{i=0}^{k} \widehat{\tilde{\theta}}_i$.

The two estimators $\widehat{a}_{1,k}$ and $\widehat{\tilde{a}}_{1,k}$ obtained from equations (3) and (4) are consistent estimators of $a_{1,k}$ in equation (2). The consistency is a direct consequence of the properties of technology and composite non–technology shocks, since they are mutually orthogonal and serially uncorrelated. The consistency property is obtained under the assumption that hours is a stationary process.[6] Hours worked per capita are by definition bounded and therefore the stochastic process of this variable cannot asymptotically have a unit root even though a unit process could provide a good statistical approximation in a small sample. To derive the consistency property, only the asymptotic behavior of hours worked matters. Consequently, the consistency of the estimators $\widehat{a}_{1,k}$ and $\widehat{\tilde{a}}_{1,k}$ for both specifications is derived under the assumption that hours worked per capita is a stationary process.[7] This property of both estimators implies that the specification of hours (level or first difference) does not asymptotically matter for the estimation of the effect of a technology improvement on this variable.[8] However, the small sample behavior of the estimators associated to the two specifications of our approach can differ, especially when hours display high persistence. We will investigate this issue in the next section.

2 Testing the measurement device

This section provides simulation experiments from an DSGE model estimated with US data.[9] The model allows for habits in consumption and investment adjustment costs. Both mechanisms have proven useful in accounting for the dynamics of macroeconomic variables in particular in terms of their persistence properties (see e.g. Beaudry and Guay, 1996, Boldrin, Christiano and Fisher, 2001 and Christiano, Eichenbaum and Evans, 2005).

Intertemporal consumption choices are not time separable and the flows of consumption services are a linear function of current and lagged consumption decisions. The intertemporal expected utility function of the representative household is given by

$$E_t \sum_{i=0}^{\infty} \beta^i \{\log(c_{t+i} - bc_{t+i-1}) + \chi_{t+i}\psi \log(1 - h_{t+i})\}$$

where $\beta \in (0, 1)$ is the discount factor, $b \in [0, 1)$ rules the degree of habit persistence and $\psi \geq 0$ is a scale parameter. E_t denotes the expectation operator conditional on the information set at period t. The variables c_t and h_t represent consumption and labor supply at time t. Time endowment is normalized to one for every period. The labor supply is subjected to a preference shock χ_t, which follows a stationary stochastic process

$$\log(\chi_t) = \rho_\chi \log(\chi_{t-1}) + \sigma_\chi \varepsilon_{\chi,t}$$

where $|\rho_\chi| < 1$, $\sigma_\chi > 0$ and $\varepsilon_{\chi,t}$ is iid with zero mean and unit variance. As noted by Galí (2005), this shock represents a sizeable source of aggregate fluctuations as it accounts for persistent shifts in the marginal rate of substitution between goods and work. Moreover, it captures different distortions on the labor market, labeled *labor wedge* in Chari, Kehoe and McGrattan (2007).

The representative firm produces a homogeneous final good y_t by means of capital, k_t, and labor, h_t, using a constant returns–to–scale technology represented by the following Cobb–Douglas production function

$$y_t = k_t^\alpha (z_t h_t)^{1-\alpha}$$

where $\alpha \in (0,1)$. z_t is a shock to total factor productivity and is assumed to follow a random walk process with drift of the form

$$\log z_t = \gamma_z + \log z_{t-1} + \sigma_z \varepsilon_{z,t}$$

where $\sigma_z > 0$ and $\varepsilon_{z,t}$ is iid with zero mean and unit variance. The constant term $\gamma_z > 0$ is the drift term and accounts for the deterministic component of the growth process. The homogenous good can be used for consumption c_t and investment x_t purposes. Capital accumulation is governed by the following law of motion

$$k_{t+1} = (1-\delta)k_t + \left[1 - S\left(\frac{x_t}{x_{t-1}}\right)\right] v_t x_t$$

where $\delta \in (0,1)$ is the constant depreciation rate and $S(.)$ reflects the presence of adjustment costs. We assume that $S(.)$ satisfies (i) $S(\gamma_z) = S'(\gamma_z) = 0$ and (ii) $\xi = S''(\gamma_z)\gamma_z^2 > 0$.

It follows that the steady state of the model does not depend on the parameter ξ while its dynamic properties do. As in Smets and Wouters (2007), the variable υ_t represents a disturbance to the investment–specific technology process and is assumed to follow a first order autoregressive process

$$\log(\upsilon_t) = \rho_\upsilon \log(\upsilon_{t-1}) + \sigma_\upsilon \varepsilon_{\upsilon,t}$$

where $|\rho_\upsilon| < 1$, $\sigma_\upsilon > 0$ and $\varepsilon_{\upsilon,t}$ is iid with zero mean and unit variance. Finally the market clearing condition on the good market writes: $y_t = c_t + x_t$.

As usual, the model is deflated for the stochastic trend component z_t and log-linearized around the deterministic steady state to obtain a state-space representation. Let Ψ denotes the whole set of model parameters. The parameters of the state-space solution of the model depends on complicated nonlinear functions of Ψ. We split Ψ in two vectors Ψ_1 and Ψ_2. The first vector $\Psi_1 = \{\beta, \alpha, \delta, \gamma_x, \psi\}$ includes parameters which are calibrated for the US economy prior to estimation. The discount factor β is chosen such that the steady–state annual return to capital equals 3.6%. The elasticity of output to the labor input $1-\alpha$ equals 0.67, which corresponds to the average share of labor income to output. The depreciation rate of physical capital δ and the gross growth rate of total factor productivity γ_x are set equal to 0.0153 and 0.0040, respectively. The value of ψ in the utility function is chosen such that households allocate 20% of their time to market activities. All theses values are reported in the first column of Table 1. The second vector $\Psi_2 = \{b, \xi, \sigma_z, \rho_\chi, \sigma_\chi, \rho_\upsilon, \sigma_\upsilon\}$ contains the parameters which summarize the law of motion of the three forcing variables $(\sigma_z, \rho_\chi, \sigma_\chi, \rho_\upsilon, \sigma_\upsilon)$, the habit persistence (b) and adjustment costs on investment (ξ).

From the state–space representation resulting from the log-linearized version of the model and under the assumption of Gaussian shocks, the log–likelihood function can be evaluated. The parameters of vector Ψ_2 are then estimated by maximizing this function. We use US quarterly data covering the sample period 1948Q1–2003Q4. The observed variables are: the growth rate of per capita output $\Delta \log y_t$, the consumption to output ratio $\log c_t - \log y_t$ and the investment to output ratio $\log x_t - \log y_t$.[10] The vector of observations is centered prior to estimation and is assumed stationary.[11] The estimation results are reported in the second column of Table 1.

The parameters are precisely estimated and are close to previous estimates for the US economy (see Smets and Wouters, 2007). The habit b and the adjustment cost ξ parameters take large values (0.6397 and 7.9894, respectively), consistent with previous estimations. These estimated values are crucial in replicating US data. For example, setting $b = \varphi = 0$ dramatically reduces the log–likelihood and a likelihood ratio test strongly rejects this restriction. In other words, our estimation results favor a version of the model with a sizeable amount of real frictions. The estimated value of σ_z slightly exceeds previous findings on US data (see Erceg, Guirieri and Gust, 2005, Chari, Kehoe and MacGrattan, 2008, among others), but values around 1.5% are not formally rejected. The autoregressive parameter ρ_χ on the preference shock is large (0.97). This empirical finding is also in line with previous research which reports that this forcing variable in estimated general equilibrium models generally displays a high degree of serial correlation (see Christiano, Eichenbaum and Vigfusson, 2006 and Chari, Kehoe and MacGrattan, 2008, among others). The investment shock exhibits less persistence but its standard error is significantly higher than the one for the permanent technology shock.

From these estimated values, we compute the dynamic responses of hours worked implied by the model (see the solid line in Figure 1). Hours worked decrease on impact and its response turns out to be positive after one year. These findings are again in accordance with those obtained from estimated DSGE models (see Smets and Wouters, 2007), from SVAR models (see Galí, 1999, Francis and Ramey, 2008) and direct measures of TFP (see Basu, Fernald and Kimball, 2006). In our model, this response of hours is the result of the interplay between habit persistence in consumption and adjustment costs on investment. As pointed out by Francis and Ramey (2005), strong enough habit persistence induces a sluggish response of consumption. Facing a positive technology shock, households can put the extra resources on investment. However, the high degree of adjustment cost on capital implies that an additional investment is very costly. Consequently, households choose to spend their new wealth on the only remaining choice, *i.e.* they increase their leisure. We also use the estimated DSGE model in order to compute other statistics which summarize the time series behavior of hours and the consumption to output ratio (in logs). First, we evaluate the contribution of the technology shock. It appears that this shock accounts for a tiny portion of fluctuations in hours worked since it represents 5.85% of their variance. At the same time, this shock represents one third of the volatility of the consumption to output ratio. These findings are in accordance with previous DSGE estimates with US data (see Smets and Wouters, 2007). The computation of the autocorrelation function of hours and the consumption to output ratio between 1 and 15 (not reported here to save space) show that the consumption to output ratio displays significantly less persistence than hours.

We now use the model to simulate artificial data, over which we replicate the different structural VARs used in the relevant literature and in the empirical part of the paper. To compute artificial time–series of the variables of interest, we draw $S = 1000$ independent random realizations of the TFP innovation $\varepsilon_{z,t}$, the labor supply shock innovation $\varepsilon_{\chi,t}$ and the investment shock innovation $\epsilon_{\upsilon,t}$. Using the parameters of Table 1, we compute $S = 1000$ equilibrium paths for the growth rates of labor productivity, of hours worked (in level and in first difference) and the log of the consumption to output ratio. In all experiments, the sample size is equal to 224 quarters, as in actual data. In order to reduce the influence of initial conditions, the simulated sample includes 250 initial points which are subsequently discarded before the estimation of VAR models. For each draw, the number of lags in VAR models is set to 4, a value typically used in empirical studies. In order to evaluate the relative performance of the different approaches, we compute the cumulative absolute bias and Root Mean Square Error (RMSE).[12]

The results are reported in Table 2 and in Figure 1. Let us first consider the DSVAR model with two variables. The response of hours obtained from this model displays a large downward bias (see panel (a) of Figure 1), and it is persistently negative. This result is similar to Chari, Kehoe and McGrattan (2008) who obtain that the difference specification of hours can create distortions and can lead to biased estimated responses under a DGP with stationary hours. Of all our experiments, this specification delivers the worst results. The responses of hours obtained from a LSVAR model displays a large upward bias, as the estimated response on impact is almost twice the true response and is persistently above the true response (see panel (a) of Figure 1). In addition, the confidence intervals (not reported) with the LSVAR model are very large and therefore not informative. This result is reflected in large absolute bias and RMSE (see Table 2).

We now consider a three–variables version of the DSVAR model. The results show improvement. The bias is reduced (see panel (b) of Figure 1 and Table 2). In particular, the DSVAR model replicates very well the response of hours on impact but diverges after horizon 5. These findings can be explained as follows. First, including the consumption to output ratio may help to separate transitory from permanent components, as argued by Cochrane (1994). Second, the first difference specification allows to remove low frequency components in hours worked. However, when the horizon increases, the DSVAR model does not properly uncover the true response since it displays a permanent effect of technology shock on hours. This finding arises because hours are over–differentiated. For the LSVAR specification, the results with three variables are similar to those with two variables. Notably, this model still over–estimates the true response. A possible explanation is that hours in level will contaminate the identification of the technology shock since this specification implies that hours still contains low frequency movements. Although reduced in the short run, the discrepancy between DSVAR and LSVAR models is maintained when the horizon increases.

Finally, we report in Table 2 and in Figure 1 the simulation results with the two–step approach. For comparison purpose, Table 2 reports the reduction in cumulated absolute bias and RMSE delivered by the two–step approach with hours in first difference. A positive value means that the two–step procedure with a first difference specification of hours delivers smaller bias (Absolute Bias and RMSE) than the other approaches. A negative value means the reverse. Panel (c) of Figure 1 displays the two estimated responses. As this figure shows, the specification of hours has little effect on the estimated responses since no conflict between the two estimated responses appears. This is confirmed by the cumulative absolute bias, which is very similar for the two specifications of hours. The two–step approach delivers the smallest cumulated absolute bias, with the exception of the DSVAR model with three variables in the very short run. In most cases, the two–step approach greatly improves the estimated dynamic responses of hours. For example, the improvement for the cumulated absolute bias is of the order of 43% and 31% compared to the LSVAR and DSVAR models with three variables, respectively.[13] At the same time, the difference appears relatively small between the two specifications of hours in the two–step approach (around 11%). The two–step approach with hours in first difference provides also the smallest cumulative RMSE when the horizon increases. According to the cumulative absolute bias and RMSE, the specification in first difference in the second step yields more precise estimates of the dynamic response than the level specification. This result suggests that the specification in difference must be preferred. All our findings illustrate previous arguments: *(i)* the consumption to output ratio allows to properly separate permanent from transitory components in labor productivity and thus identify permanent technology shocks, *(ii)* hours must be excluded from the SVAR because they contaminate the identification of permanent shocks.

As pointed out by Chari, Kehoe and Mc Grattan (2008) and Christiano, Eichenbaum and Vigfusson (2006), the simulation results crucially depend on the relative size of shocks (permanent/transitory) and their persistence. We now investigate these two quantitative issues. First, we set the three standard–errors of shocks according to

$$\sigma_z = \widehat{\sigma}_z \equiv 0.0217 \quad , \quad \sigma_\chi = \tau \times \widehat{\sigma}_\chi \equiv \tau \times 0.0249 \quad \text{and} \quad \sigma_\upsilon = \tau \times \widehat{\sigma}_\upsilon \equiv \tau \times 0.2374$$

where $\tau \in [0.1, 0, 2]$. The standard–error of the technology shock remains unchanged, whereas the two others will vary between (almost) zero and two times their estimated values. When $\tau = 2$, the variance of hours explained by the technology shock is very small (1.53%), whereas it represents a non–negligible portion of the consumption to output ratio (10.71%). Conversely, when $\tau = 0.1$, these statistics are equal to 86.13% and 97.96%, respectively. For each selected value of $\tau \in [0.1, 2]$, we simulate artificial data, estimate the dynamic responses with each approach and compute the cumulated absolute error between horizon 0 and 12. The results are reported in panel (a) of Figure 2. This sensitivity analysis shows that the previous results are left unaffected.[14] When the standard–errors of the two non–technology shocks are small, the bias is reduced with both approaches. When these standard–errors increase, the bias increases but again the two–step approach (both with a level and first difference specification of hours) delivers the smallest bias. Second, we modify the persistence of stationary shocks in the model. More precisely, we inspect the role played by the highly persistent preference shock (recall that $\widehat{\rho}_\chi = 0.97$). Panel (b) of Figure 2 reports the cumulative absolute bias between horizon 0 and 12 when ρ_χ varies between 0.9 and 0.99. As shown in this figure, the two–step approach is not very affected by the persistence of the preference shock, since its cumulative absolute bias remains almost constant. This is not the case with LSVAR and DSVAR models. Again, our two–step approach outperforms standard SVARs.

3 Empirical Results

We now apply the two-step methodology to US data. Except for the Federal Fund rate, the data cover the sample period 1948Q1-2003Q4. We consider different measures of hours and output, bivariate VARs and larger VAR specifications, different sample periods and breaks in labor productivity.

We first present results based on a simple bivariate VAR model in the first step. This VAR model includes the growth rate of labor productivity and the log of consumption to output ratio. Labor productivity is measured as the non farm business output divided by non farm business hours worked. Consumption is measured as consumption on non-durables and services and government expenditures. The ratio is obtained by dividing these nominal expenditures by nominal GDP. In the second step, the log level h_t (see equation (3)) and the growth rate of hours Δh_t (see equation (4)) are projected on the estimated technology shocks. Hours worked in the non farm business sector are converted to per capita terms using a measure of the civilian population over the age of 16. The period is 1948Q1-2003Q4 and we will therefore refer to this as the long sample.

We also compare the estimations results with our two–step approach to those obtained from the estimation of SVAR models. As a benchmark, these SVAR models include growth rate of labor productivity and either the log level of hours (LSVAR) or the growth rate of hours (DSVAR). We have also investigated larger LSVAR and DSVAR models. In each of the SVAR models, we identify technology shocks as the only shocks that can affect the long-run level of labor productivity. The lag length p for each VAR model (1) is obtained using the Hannan–Quinn criterion. For each estimated model, we also apply a LM test to check for serial correlation. The number of lags p is 3 or 4 depending on the data and the sample. In the second step, we include the current and past twelve values of the identified technology shocks in the first step, *i.e.* $q = 13$ in equations (3) and (4).

In order to assess the dynamic properties of hours worked and consumption to output ratio (in logs), we first compute their autocorrelation functions (ACFs). Figure 3 reports these ACFs for lags between 1 and 15. As this figure makes clear, the autocorrelation functions of hours worked always exceed those of the consumption to output ratio and decay at a slower rate. Additionally, we perform Augmented Dickey Fuller (ADF) test of unit root. For each variable, we regress the growth rate on a constant, lagged level and four lags of the first difference. The ADF test statistic is equal to -2.74 for hours and -2.93 for the consumption to output ratio. This hypothesis cannot be rejected at the 5 percent level for hours, whereas it is rejected at the 5 percent level for the consumption to output ratio. The ACFs and the ADF test suggest that this latter variable is less persistent than hours.

The estimated IRFs of hours after a technological improvement are reported in Figure 4. The upper left panel shows the well known conflicting results of the effect of a technology shock on hours worked between LSVAR and DSVAR specifications. The LSVAR specification displays a positive hump–shaped response whereas the DSVAR specification implies a decrease in hours. We obtain wide confidence intervals (not reported) in the LSVAR specification, such that the estimated IRFs of hours are not significantly different from zero at any horizon. For the DSVAR specification, the impact response is significant, but as the horizon increase the negative response is not significantly different from zero. The conflicting result between LSVAR and DSVAR specifications is virtually unaffected if these specifications include the log of the consumption to output ratio together with the growth rate of labor productivity and the log (level or first difference) of hours (see Figure 5 in appendix). In SVARs, the consumption to output ratio does not help to reconcile the two specifications.[15]

In contrast, the two-step approach delivers almost the same picture whether hours are specified in level or first difference (see the upper right panel of Figure 4). In the very short run, the IRFs of hours are identical and when the horizon increases the positive response is a bit more pronounced when hours are taken in level rather than in first difference. On impact, hours worked decrease, but after five periods the response becomes persistently positive and hump–shaped.

The bottom panel of Figure 4 also reports the 95 percent asymptotic confidence interval. The confidence interval is wide when we consider hours in level. Consequently, these responses cannot be used, for instance, to discriminate among business cycle theories. In contrast, when hours are projected in first difference, the dynamic response are very precisely estimated. On impact, hours significantly decrease. Moreover, the positive hump–shaped response after 8 quarters is precisely estimated. Notice that these findings are in accordance with simulation experiments of section 2. Our empirical results are also in line with those of previous empirical papers which obtain that hours fall significantly on impact (see Galí, 1999, Basu, Fernald and Kimball, 2006, Francis and Ramey, 2005, 2008), but display a hump–shaped positive response during the subsequent periods (see Vigfusson, 2004).

We now check the robustness of our first results to different measures of hours and output, bivariate VARs and larger VAR specifications, different sample periods and breaks in labor productivity. The results are reported in Figures (6)–(11).

We first consider an alternative measure of output (labor productivity) and hours with the long sample. The alternative measure of productivity and hours is based on business sector data. Figure 6 shows that the IRFs are similar to those reported in Figure 4, especially for hours worked in first difference. Hours decrease in the short run but increase after four quarters. While the shape of the IRFs is similar for both specifications, the estimated values differ more than the ones obtained with non farm business sector data. To understand this difference, Figure 7 in the Appendix reports the estimated response of hours from the LSVAR and DSVAR specifications for both data sets: non farm business data and business sector data. Although the DSVAR specification delivers the same response for both sets of data, the positive estimated response from LSVAR specification for the business sector is almost three times larger than for the non farm business sector. The difference between the response of hours from the LSVAR and DSVAR specifications is then exacerbated for the business sector data.

We now maintain the bivariate SVAR model in the first step but replace the log of the consumption to output ratio by the log of the ratio of nominal investment expenditures to nominal GDP. Investment is measured as expenditures on consumer durables and private investment. This ratio is another promising candidate in the SVAR model, since it displays lower serial correlation than hours. Indeed, Figure 3 shows that the ACFs of the ratio are substantially lower than the ones of hours for any lag. These ACFs are very similar to the ones for the consumption to output ratio. In addition, we perform ADF test of unit root including four lags and a constant term. The ADF test statistic is equal to -3.50 for the investment to output ratio. The null hypothesis of unit root is rejected at the 1 percent level. We consider again non farm business data and the long sample.[16] Figure 8 displays the IRFs. The replacement of consumption to output ratio by the investment to output ratio does not modify the previous findings and the response of hours displays the same pattern. The two specifications yield very similar IRFs for hours and again the confidence intervals are wide when hours are considered in level. Notice that the negative impact response in not significantly different from zero with hours in first difference. Moreover, the positive hump–shaped pattern of hours is precisely estimated.

We now examine the robustness of the two-step strategy using a larger VAR system in the first step. We maintain non farm business data for labor productivity and hours and we use the long sample. The SVAR model in the first step includes labor productivity growth, consumption to output ratio, investment to output ratio and the rate of inflation. The measure of inflation is obtained using the growth rate of the GDP deflator. Results are reported in Panel (a) of Figure 9. The IRFs are very similar to those of Figure 4. Moreover, IRFs are close for both specifications. Again the specification with hours in difference in the second step delivers precise estimates of the IRFs: hours significantly decrease in the short–run, but positively increases after two years. Conversely, the confidence interval with hours in level is so wide that results obtained with this specification are not very informative.

Using this larger system, the exercise is repeated with a shorter sample. Since much of business cycle literature is concerned with post–1959 data, we follow Christiano, Eichenbaum and Vigfusson (2004) and therefore consider a second sample period given by 1959Q1–2003Q4. Panel (b) of Figure 9 reports the estimated responses. We obtain again the same shape for the IRFs previously obtained from a level and a first difference specification of hours. The negative responses in the short–run differ slightly according to the specification of hours, but the two IRFs become positive and very close after five periods. The difference in the two IRFs can be explained by the higher persistence of the hours series for this shorter sample. Indeed, the ADF test statistic is equal to -2.47 for the short sample compared to -2.74 for the long sample. Again, the response of hours is precisely estimated when hours are taken in difference. This is not the case for the level specification which appears less informative.

We also add the federal fund rate in the larger system and consider the short sample 1959Q1–2003Q4. The results are reported in Figure 10. The negative response of hours is more pronounced in the short run compared to the previous cases (when hours are taken in first difference), but we still find a persistent increase in the subsequent periods. Notice that the response of hours differs according to their specification, but the shapes of the two IRFs remain very similar. As for other cases, the confidence intervals for the level specification are larger but the difference in the confidence intervals between both specifications is here amplified.

As last experiment, we investigate the sensitivity of our results to structural breaks in labor productivity. We consider this issue in the context of the latter experiment. Fernald (2007) shows that once we allow for trend breaks in labor productivity, the response of hours to a technology shock in the LSVAR model becomes persistently negative.[17] The breaking dates identified by Fernald are 1973Q1 and 1997Q2. Labor productivity growth is first regressed on a constant, a pre–1973Q1 dummy variable and a pre–1997Q1 dummy variable. The residuals of this regression are then used as a new measure of labor productivity growth in the first step. The responses of hours are reported in Figure 11. The response appears unaffected as the negative response on impact is around -0.2 (see Figure 10 for a comparison). Moreover, the hump–shaped and delayed–positive response is maintained for both specifications and is significant for the specification in difference. A possible explanation of the robustness to potential breaks is the following. The response of hours to a technology shock in the LSVAR specification is sensitive to time variations, *i.e.*

breaks in labor productivity. These breaks alter the low frequency correlation between hours and labor productivity, but does not modify the one between consumption to output ratio and labor productivity. Since hours are eliminated from the VAR model in the first step, our approach seems to be more immune to structural time variations.

Finally, Figure 12 compares the dynamic responses of hours worked for all cases examined above. The results when hours are specified in level are reported in the left panel of this Figure, while the ones with a specification of hours in first difference are in the right panel. As this figure shows, the dynamic responses of hours in all cases and for both specifications are remarkably similar. In the very short–run, hours decrease after a technology improvement. After some period, hours gradually increase and display a hump–shaped pattern. This finding does not vary too much with different sample periods, variables included in the VAR model at the first step and structural breaks in labor productivity.[18] That seems to confirm the robustness of our proposed two-step strategy and the appeal of this alternative simple approach for further empirical investigations.

4 Concluding Remarks

This paper uses a simple two-step approach to consistently estimate technology shocks and the responses of hours worked after a technology improvement. In a first step, a SVAR model with labor productivity growth and the log of consumption to output ratio allows to identify and estimate technology shocks. In a second step, the response of hours is obtained by a simple regression of hours on the estimated technology shocks. Simulation experiments conducted from an estimated DSGE of the US economy show that the two–step procedure outperforms LSVARs and DSVARs. We obtain that the consumption to output ratio helps a lot to separate permanent from transitory components in labor productivity. The two-step approach, when applied to US data, predicts a short–run decrease of hours after a technology improvement, as well as a delayed and hump–shaped positive response. The dynamic responses of hours are precisely estimated with a first difference specification, whereas their confidence intervals are wide with a level specification. These findings appear robust to different sample periods, measures of hours and output and to the variables included in the VAR model in the first step. The proposed approach is devoted here to the estimation of the responses of hours worked. However, this empirical strategy can easily be used to evaluate the effect of a technology shock on other persistent aggregate variables.

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Notes

[1]Simulation experiments based on a basic RBC model in Fève and Guay (2007) show clear evidence that the uncertainty about the right specification of hours in the SVAR model is more detrimental for the estimation of technology shocks and their impacts on hours than the information loss resulting from the omission of this variable in the SVAR model. These results are confirmed in section 2 by simulation experiments conducted from an estimated DSGE model featuring sizeable real frictions.

[2]It should be noted that we use labor productivity growth rather than output growth, as in Blanchard and Quah (1989) and Cochrane (1994). Galí(1999) shows that labor productivity growth must be preferred to output growth if there exists shocks that permanently and jointly shift the output and the labor input.

[3]As pointed out by Fernandez–Villaverde, Rubio–Ramirez and Sargent (2005), the invertibility of VARs (and related to this, the existence of a fundamental representation) is an important quantitative issue . Using the estimated DSGE model of Section 2, we check the invertibility condition for the SVAR associated to our two–step estimator. The DSGE model contains three shocks (a permanent technology shock and two stationary shocks), while the SVAR model includes only two variables in the first step. We use the innovations representation of Fernandez–Villaverde, Rubio–Ramirez and Sargent (2005) and then their general formulation to check for invertibility. We find that the SVAR model of step 1 admits an infinite autoregressive and fundamental representation.

[4]See Blanchard and Quah (1989) and Faust and Leeper (1997) for a discussion on the conditions for valid shock aggregation in the small SVAR models.

[5]In typical DSGE models, non–technology shocks correspond to preference, taxes, government spending, monetary policy shocks and so on (see Smets, and Wouters, 2007). When the number of stationary variables in the SVAR model is small respective to the number of these shocks and without additional identification schemes, these shocks are not identifiable. For our purpose, this identification issue does not matter since we only focus on the dynamic response of hours to a (permanent) technology shock.

[6]When a stationary variable is included in difference in a VAR, the resulting estimators are biased due to the impossibility of a finite VAR to properly approximate an unit root in the MA component. The second step here does not suffer from this problem because the variable in difference is directly regressed on the estimated technology shocks so it does not need to approximate a MA component by a finite autoregression.

[7]See Fève and Guay (2007) for a formal proof.

[8]The computation of the corresponding confidence intervals raises two practical econometric issues for our procedure. First, confidence intervals in the second step must account for the uncertainty resulting from the first step estimation. This is usually called the *generated regressors problem*. Second, the residuals in the second step can be serially correlated in practice. This is especially true for the regression (3) with hours in level. Confidence intervals of IRFs are computed using a consistent estimator of the asymptotic variance-covariance of the second step parameters(see Newey and West 1994). The consistent estimator that we use is borrowed from Newey (1984). Indeed, our two step procedure can be represented as a member of the method of moments estimators. With this representation in hand, we can derive the asymptotic variance-covariance matrix of the second step estimator (see Fève and Guay 2007 for more details).

[9]See Erceg, Guerrieri and Gust (2005), Chari, Kehoe and Mc Grattan (2008) and Christiano, Eichenbaum and Vigfusson (2006) for other simulation experiments.

[10]See section 3 for more details.

[11]Unit root tests conducted in section 3 indicate that the null hypothesis (of a unit root) for the two ratios is rejected at conventional levels.

[12]The cumulative absolute bias at horizon k is defined as $\sum_{i=0}^{k} |irf_i(model) - irf_i(svar)|$ where $irf_i(model)$ denotes the model's impulse response and $irf_i(svar) = (1/N)\sum_{j=1}^{N} irf_i(svar)^j$ the mean of impulse responses over the N simulation experiments obtained from SVARs. The cumulative RMSE at horizon k is defined as $\sum_{i=0}^{k} rmse_i$ where $rmse_i = ((1/N)\sum_{j=1}^{N}(irf_i(model) - irf_i(svar)^j)^2)^{1/2}$ represents the RMSE at horizon i.

[13]The improvement is of order of 75% and 400% compared to the LSVAR and DSVAR

models with two variables, respectively.

[14]For the readability of this figure, we do not report the results with the two–variable LSVAR and DSVAR models, given their relative poor performances.

[15]Christiano, Eichenbaum and Evans (2005) also obtain conflicting results in larger SVARs. Furthermore, we have considered a six–variable DSVAR and LSVAR models and we still find opposite results for the two specifications. The six–variable SVAR includes labor productivity growth, hours (level or difference), consumption to output ratio, investment to output ratio, the inflation rate and the Federal Fund rate. The data concern Non Farm Business Sector and the sample Period is 1959Q1–2003Q4.

[16]We obtain similar results (not reported) with business sector output and hours.

[17]Gambetti (2005) finds similar results in a Time–Varying Coefficients Bayesian VARs.

[18]One exception concerns the dynamic responses with Business Sector data and the level specification of hours. However, the results with the difference specification is not sensitive to the measure of output (labor productivity) and hours. Notice that this result is already present in SVARs.

Table 1: Parameter values

Calibrated		Estimated		
Parameter	Value	Parameter	Value	s.e.
β	0.9950	b	0.6397	0.0650
α	0.3300	ξ	7.9894	1.3076
δ	0.0153	σ_z	0.0217	0.0079
γ_z	0.0040	ρ_χ	0.9700	0.0219
ψ	3.6295	σ_χ	0.0249	0.0072
		ρ_υ	0.3363	0.0846
		σ_υ	0.2374	0.0530

Note: US quarterly data covering the sample period 1948:1–2003:1. The vector of observed data includes GDP, consumption to output ratio and investment to output ratio.

Table 2: Simulation Results

Horizon	LSVAR 2 Variables	DSVAR 2 Variables	LSVAR 3 Variables	DSVAR 3 Variables	Two Step Level	Two Step First Difference
			Cumulative Absolute Bias			
0	0.6473	0.5086	0.6422	0.4795	0.7510	0.4997
4	3.0821	4.5693	2.7529	0.9846	2.3370	1.1202
8	4.6277	9.8806	3.9457	2.0426	2.6518	1.8627
12	5.4563	15.5743	4.4814	4.0906	2.7823	3.1165
			Reduction in Cumulative Absolute Bias (in %)			
0	29.54	1.78	28.52	-4.04	50.29	–
4	175.14	307.90	145.76	-12.11	108.63	–
8	148.44	430.45	111.83	9.66	42.36	–
12	75.08	399.74	43.80	31.26	-10.72	–
			Cumulative RMSE			
0	0.9842	0.5459	0.8703	0.7168	1.1513	0.7549
4	2.3843	2.2785	2.1213	1.5147	2.2179	1.4238
8	3.0456	3.6383	2.7699	2.0804	2.7272	1.8712
12	3.4217	4.7326	3.1611	2.6244	3.1210	2.3210
			Reduction in Cumulative RMSE (in %)			
0	30.37	-27.69	15.29	-5.05	52.51	–
4	67.46	60.03	48.99	6.38	55.77	–
8	62.76	94.44	48.03	11.18	45.75	–
12	47.42	103.90	36.20	13.07	34.47	–

Note: DSVAR, LSVAR and two–step identification. The LSVAR–2 Variables model includes labor productivity growth and the log of hours. The DSVAR–2 Variables model includes labor productivity growth and the log of hours in first difference. The LSVAR–3 Variables model includes labor productivity growth, the log of hours and the log of consumption to output ratio. The DSVAR–3 Variables model includes labor productivity growth, the log of hours in first difference and the log of consumption to output ratio. For the two–step procedure. the SVAR model in the first step includes labor productivity growth and the log of consumption to output ratio. In the second step, the dynamic responses of hours are obtained from hours in level (Two–Step Level) and in first difference (Two–Step Difference). Reduction in Cumulative Absolute Bias and in Cumulative RMSE (in %) are obtained using the Two–Step approach with hours in difference as the reference. A positive value means that the Two–Step with a first difference specification of hours delivers smaller bias (Absolute Bias and RMSE) than the other approaches. A negative value means the reverse. Results are obtained from 1000 experiments. The sample size is equal to 224 quarters. The simulated sample includes 250 initial points which are subsequently discarded before the estimation of VAR models. The selected horizon for IRFs is 13. For each draw, the number of lags in both VAR models is set to 4.

Figure 1: True and Estimated IRFs of Hours

Figure 2: Sensitivity Analysis (Cumulative Absolute Bias)

(a) Changing the Volatility of Shocks

(b) Changing the Persistence of Shocks

Figure 3: ACFs

Note: NFB Sector data and Sample Period 1948Q1–2003Q4. All variables in logs.

Figure 4: IRFs of Hours to a Technological Improvement (NBF data)

Note: DSVAR, LSVAR and two–step identification. The DSVAR model includes labor productivity growth and the log of hours in first difference. The LSVAR model includes labor productivity growth and the log of hours. For the two–step procedure, the SVAR model in the first step includes labor productivity growth and the log of consumption to output ratio. In the second step, the dynamic responses of hours are obtained from equations (3) and (4). Top left panel, IRFs computed from DSVAR and LSVAR specifications. Top right panel, IRFs computed from two–step procedure (equations (3) and (4)). Bottom left panel, IRFs obtained with the log of hours in level in the second step. Bottom right panel, IRFs obtained with the log of hours in first difference in the second step. Non Farm Business Sector data and sample period 1948Q1–2003Q4. The selected horizon for IRFs is 13. 95 percent asymptotic confidence interval shown.

Figure 5: Three–Variables SVARs

Note: The DSVAR model includes labor productivity growth, the log of consumption to output ratio and the log of hours in first difference. The LSVAR model includes labor productivity growth, the log of consumption to output ratio and the log of hours in level. Non Farm Business Sector data and sample period 1948Q1–2003Q4. The selected horizon for IRFs is 13. Asymptotic confidence interval not reported.

Figure 6: IRFs of Hours with Business Sector Data

Note: Two–step identification. The SVAR model in the first step includes labor productivity growth and the log of consumption to output ratio. In the second step, the dynamic responses of hours are obtained from equations (3) and (4). Left panel, IRFs obtained with the log of hours in level. Right panel, IRFs obtained with the log of hours in first difference. Business Sector data and sample period 1948Q1–2003Q4. The selected horizon for IRFs is 13. 95 percent asymptotic confidence interval shown.

Figure 7: SVARs with Different Sector Data

Note: The DSVAR model includes labor productivity growth and the log of hours in first difference. The LSVAR model includes labor productivity growth and the log of hours in level. Business Sector data, Non Farm Business Sector data and sample period 1948Q1–2003Q4. The selected horizon for IRFs is 13. Asymptotic confidence interval not reported.

Figure 8: IRFs of Hours using Investment to Output Ratio

Note: Two–step identification. The SVAR model in the first step includes labor productivity growth and the log of investment to output ratio. In the second step, the dynamic responses of hours are obtained from equations (3) and (4). Left panel, IRFs obtained with the log of hours in level. Right panel, IRFs obtained with the log of hours in first difference. Non Farm Business Sector data and sample period 1948Q1–2003Q4. The selected horizon for IRFs is 13. 95 percent asymptotic confidence interval shown.

Figure 9: IRFs of Hours with a Four Variable System

Panel (a). NFB Sector data and Sample Period 1948Q1–2003Q4

Note: Two–step identification. The SVAR model in the first step includes labor productivity growth, the log of consumption to output ratio, the log of investment to output ratio and the rate of inflation. In the second step, the dynamic responses of hours are obtained from equations (3) and (4). Left panel, IRFs obtained with the log of hours in level. Right panel, IRFs obtained with the log of hours in first difference. Non Farm Business Sector data and sample period 1948Q1–2003Q4. The selected horizon for IRFs is 13. 95 percent asymptotic confidence interval shown.

Panel (b). NFB Sector data and Sample Period 1959Q1–2003Q4

Note: Two–step identification. The SVAR in the first step includes labor productivity growth, the log of consumption to output ratio, the log of investment to output ratio and the rate of inflation. In the second step, the dynamic responses of hours are obtained from equations (3) and (4). Left panel, IRFs obtained with the log of hours in level. Right panel, IRFs obtained with the log of hours in first difference. Non Farm Business Sector data and sample period 1959Q1–2003Q4. The selected horizon for IRFs is 13. 95 percent asymptotic confidence interval shown.

Figure 10: IRFs of Hours with a Five Variable System

Note: Two–step identification. The SVAR model in the first step includes labor productivity growth, the log of consumption to output ratio, the log of investment to output ration, the inflation rate and the Federal Fund rate. In the second step, the dynamic responses of hours are obtained from equations (3) and (4). Left panel, IRFs obtained with log of hours in level. Right panel, IRFs obtained with log of hours in first difference. NFB Sector data and sample period 1959Q1–2003Q4. The selected horizon for IRFs is 13. 95 percent asymptotic confidence interval shown.

Figure 11: IRFs of Hours with Breaks in Labor Productivity

Note: Two–step identification. The SVAR model in the first step includes labor productivity growth, the log of consumption to output ratio, the log of investment to output ratio, the inflation rate and the Fed Fund rate. The breaking dates are 1973Q1 and 1997Q2. The new measure of labor productivity growth is obtained as the residual of the regression of the original measure on dummy variables associated to breaks. In the second step, the dynamic responses of hours are obtained from equations (3) and (4). Left panel, IRFs obtained with the log of hours in level. Right panel, IRFs obtained with the log of hours in first difference. NFB Sector data and Sample Period 1959Q1–2003Q4. The selected horizon for IRFs is 13. 95 percent asymptotic confidence interval shown.

Figure 12: Summary of the Results
NFB Sector data and Sample Period 1948Q1–2003Q4

Note: Legend: (1) see Figure 4; (2) see Figure 6; (3) see Figure 8; (4) see Figure 9, panel (a); (5) see Figure 9, panel (b); (6) see Figure 10; (7) see Figure 11.